A Theoretical Bayesian Game Model for the Vendor-Retailer Relation

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ABSTRACT
We consider an equilibrated supply chain with two equal partners, a vendor and a retailer (also called newsboy type products supply chain).
The actions of each partner are driven by profit. Given the fact that at supply chain level are specific external influences which affect the costs and concordant the profit, we use a game theoretic model for the situation, considering costs and demand.
At theoretical level, symmetric and asymmetric information patterns are considered for this situation. There are at every supply chain's level situations when external factors (such as inflation, raw-material rate) influence the situation of each partner even if the information is well shared within the chain. The model we propose considers both the external factors and asymmetric information within a supply chain.

KEYWORDS: *game theory, supply chain management, symmetric information, asymmetric information*

JEL Classification: *M19.*

INTRODUCTION

We consider a two-member decentralized (or equilibrated) supply chain that manufactures and sells newsboy-type products and comprises a downstream retailer and an upstream vendor. This relation (vendor-retailer) is also called seller-buyer (Esmaeili et al., 2009). The vendor is the manufacturer which wholesales a product to a retailer, who, in turn retails it to a consumer. In literature, the terms seller, supplier, and manufacturer are also used for the vendor. Likewise, the word buyer has been used to represent the retailer. In this paper we will use the nomenclature vendor and retailer.

Our model is based on the information the chain partners receive from inside the supply chain or outside the supply chain. The information we refer to is related to costs and demand. The costs are described using a probability distribution, while the demand is modeled using an econometric model. We have two cases for estimating the linear demand: one with a single variable, and one with two variables. We take into account the fact that the probability distribution of cost of each partner is influenced by uncertain information

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(internal or external regarding the supply chain). Considering information, we build a game theoretic model for reflecting the interests of the two partners, specifically obtaining profit.
The rest of this paper is organized as follows. The next section provides a brief survey of the related literature. The model is described in Section 3; Section 4 offers some concluding remarks.

1. RELATED LITERATURE

Before detailing similar models, there are several aspects we consider within our model, such as supply chain complexity, opposite interests of the partners, information sharing, symmetric and asymmetric information, which we consider to be the context of our model. All these aspects are further exposed.

Supply chains are the most complex systems at micro-economical level. This fact has increase the interest of economic researchers for the modeling of complex situations which appear in this field. The greatest challenge is to manage the relation between several partners; coordination is one top subject for the research.

One of the proven rules of supply chains is that better informed partners implies better performance at system's and partners' level. Examples are all around: Wal-Mart and Procter&Gamble, Toyota's supply chain, Dell's supply chain etc. Collaboration is the essence of supply chain existence. Using an instrument to measure supply chain collaboration, considering three dimensions: information sharing, decision synchronization and incentive alignment, Simatupang and Sridharan (2005) proven that the greater level of collaboration, the greater level of performance is achieved. Even if supply chain partnerships promises mutual benefits for the partners, those benefits are rarely attained due to different interests of the partners (Nagarajan & Sosic, 2008). Usually, the partners search only for their profit, they work having a local perspective and an opportunistic behavior. In this case, global profitability cannot be attained (Leng & Parlar, 2005).

Coordination and collaboration is practiced by several partners who understood the benefits, through several forms: information sharing, shared-savings contracts (Hennet & Arda, 2008; Corbett et al., 2005). Supply chain coordination is totally achieved only if the members of a decentralized supply chain behave as if they are operating in a centralized supply chain (Leng & Parlar, 2009).

Information sharing within supply chain can take several forms. Information sharing refers to the act of capturing and disseminating timely and relevant information for decision makers to plan and control supply chain operations (Simatupang & Sridharan, 2005). We can speak about symmetric and asymmetric information. Under a symmetric information pattern, the vendor and retailer have complete information on each other's operations (they share information). However, in a decentralized supply chain, the vendor and the retailer, being independent entities, have private information about various aspects of their businesses which are not common knowledge (they don't share information and we deal with asymmetric information). Some information is owned only by the vendor (manufacturing costs, materials' prices etc.), while some are known only by the retailer (information regarding market demand for example) (Esmaili & Zeephongsekul, 2009).
The context of our paper is that of complex supply chains, which perform their activities within complex market situations. Each partner has its own interest, while collaboration for

facing several uncertainties can be made using information sharing. In conclusion, each partner can have at least three types of information regarding its activity (whether we are speaking about costs or revenue, which together influence the profit): external information – information which does not depend on the supply chain, it is external generated and it cannot be obtained within the supply chain – we refer to information concerning prices, inflation, demand etc.; shared internal information – this type of information depends on the collaboration of supply chains' partners, some chains have it due to the practices which were implemented within, some chains don't have it due to partners' lack of common view regarding possible benefits from collaboration; individual information – this type of information refers to internal aspects concerning each partner of the supply chain and it is not shared within the supply chain. The individual information is also called unshared internal information. We consider within the model both internal information (shared or individual) and external information.

The real truth is that even if supply chain's partners are able to share their information, there still is uncertain information which influences their decisions. These include demand and supply uncertainty due to the use of unskilled labor, as well as the sudden breakdown of production facilities of upstream players (Ryu et al., 2009). Xiao and Qi (2008) disregard the general stable environments for which models within supply chain are commonly built and consider that several disruptions are not covered. Promotion of sales, raw materials shortage, new tax or tariff policies, machine breakdown are generally called production costs disruptions. We consider this aspect and build a model which takes into account this uncertain character of information.

For better understanding the progress our model brings, we shall further describe its place within the literature. Leng and Parlar (2005) made a review of several articles which used game theory in supply chain modeling. Their classification of game-theoretical applications in supply chain management is based on five application areas: inventory games with fixed unit purchase cost (1), inventory games with quantity discounts (2), production and pricing competition (3), games with other attributes (such as capacity, service/product quality, advertising and new product introduction) (4), games with joint decisions on inventory, production/pricing, and other attributes (5). Our model can be classified into the fifth category established by Leng and Parlar. It can be used for all supply chain game theory problems given the fact that it gives a new view regarding uncertainty within supply chains.

There are several closed models which we have considered while realizing our model:
Chu and Lee (2006) have built a Bayesian game for a vendor and a retailer, for modeling information sharing. They also use information signals regarding the market demand and try to model the conditions that will influence information sharing by the retailer. The retailer reveals the information if the cost of sharing the information is small and if a high demand is signaled. This model is based on information signals, but it doesn't cover the full span of the problem, it reveals only details regarding information sharing between the two partners. It is specific to this article the fact that the authors do not take information sharing for granted.
A specific thing of our article is the fact that we do not consider information sharing sufficient for being able to find chain equilibrium. We consider that external information is also a factor which should be considered in this situation.

2. THE MODEL

We consider a two-echelon supply chain, containing a downstream retailer denoted by R and an upstream vendor denoted by V. Within this chain, R and V are generally responsible about market demand. We assume that any information that R and V receive is internal or external related to the supply chain. The costs of R, denoted by c_r , and the costs of V denoted by c_v are considered their own information. The access to the information the other has is considered probabilistic. The uncertain information influences the costs, while the costs influence the prices of R and V:

$$uncertain\ information\ => \text{costs} => \ prices$$

We shall study how the uncertain information affects the prices of R and V. The hypotheses we take into account is that both R and V search for maximizing their income.
An incomplete information game is proper for modeling the exposed problem. For a detailed definition of Bayesian equilibrium, we refer the reader to Fudenberg & Tirole (1991).

Costs variation is specified using a probability two-dimensional distribution conditioned by an information source. We shall denote by Θ_1 the set of possible costs c_v for the vendor and Θ_2 the set of possible costs c_r for the retailer. In our exposition, we assume type sets Θ_i are finite, $\Theta = \Theta_1 \times \Theta_2$ is a finite set also. Let us denote by $\mu_t(\theta)$,$\theta \in \Theta$, the probability that type combination $\theta = (\theta_1, \theta_2) \in \Theta$ will be chosen at a t moment. We assume, without loss of generality, as in (Harsanyi, 1967) that players have incomplete information about their opponents' payoffs, but have complete information about strategies of all other players. In our case the prices are the strategies of the two players, R and V.

We assign a source of information for the external uncertain information of the supply chain (Parpucea & Pârv, 2011). An information source is a way of specifying the states of a process, regarding one or several variables. We shall denote by S^X the information source assigned to variable X . The set of distinct values $x_k \in X$,$k = \overline{1,n}$, represents a complete space of events. We assign to each x_k a state denoted by s_x^k. Let's assume that p_x^k is the probability that s_x^k occurs. If the information source S^X has n states, the probability of states at a t moment form a discrete variable denoted by:

$$S_t^X\colon \begin{pmatrix} s_x^k \\ p_x^k(t) \end{pmatrix}_{k=\overline{1,n}} \tag{1}$$

The probabilities $p_x^k(t)$ are an estimation of the appearance of different states for the information source for a period t. In order to simplify the presentation, a source of information shall be denoted by S_t.
Let us consider now the probability distribution μ_t defined on the discrete set Θ and an information source S_t at t moment, common to all players. According to the probability distribution conditioned by an information source, we have:

$$P(\mu_t = \theta, S_t = s^k) = P(S_t = s^k) \cdot P(\mu_t = \theta / S_t = s^k) \tag{2}$$

where $\theta \in \Theta$ and $s^k \in S_t$.

The following notations are introduced: $P(\mu_t, S_t)$ for the probability of μ_t and S_t occurring simultaneously, $P(\mu_t / S_t)$ for the probability of μ_t occurring conditioned on S_t having occurred (i.e. the conditional probability of μ_t given S_t), and $P(S_t)$ for the probability distribution of S_t. $P(\mu_t, S_t)$ is referred to as the historic probability distribution, $P(S_t)$ as the probability distribution of information source at a t moment and $P(\mu_t / S_t)$ is the posterior probability distribution. Posterior means historical updated with information. The probability distribution $\mu_t$conditioned on the information source S_t, denoted by μc_t is the probability distributionμ_t updated by the S_t.

Firstly, we present a less complex model. It is a decision making problem, used for finding the optimum prices for both supply chain partners, in accordance to the available information.

We shall denote by p_r and p_v the prices for R and V, respectively. In order to simplify the presentation we shall consider two significant values, different for each cost. The sets of possible types we assume to be:

$$\Theta_1 = \{c_v^1, c_v^2\},\ \Theta_2 = \{c_r^1, c_r^2\} \tag{3}$$

This means that for V there are two possible costs alternatives, c_v^1 and c_v^2, while for R there are also two costs alternatives c_v^1 and c_v^2. We consider that each partner performs its own market research, and they identify the demand functions denoted by:

$$q_v = a_v + b_v \cdot p_v \tag{4}$$

$$q_r = a_r + b_r \cdot p_r \tag{5}$$

where a_v, b_v , a_r , b_r represent the demand parameters for V and R .

For a state $s^k$of source S_t we built two marginal distributions μc_t^1 and μc_t^2 for the costs of V and R. The payoff functions are:

$$\pi_v(p_v, c_v^i) = (a_v + b_v \cdot p_v) \cdot (p_v - c_v^1 \cdot \mu c_t^1(c_v = c_v^1, S_t = s^k) - c_v^2 \cdot \mu c_t^1(c_v = c_v^2, S_t = s^k)) \tag{6}$$

$$\pi_r(p_r, c_r^i) = (a_r + b_r \cdot p_r) \cdot (p_r - c_r^1 \cdot \mu c_t^2(c_r = c_r^1, S_t = s^k) - c_r^2 \cdot \mu c_t^2(c_r = c_r^2, S_t = s^k)) \tag{7}$$

For V we have:

$$\pi_v(p_v, c_v^i) = b_v \cdot p_v^2 + (a_v - \bar{c_v} \cdot b_v) \cdot p_v - \bar{c_v} \cdot a_v \tag{8}$$

where:

$$\bar{c_v} = c_v^1 \cdot \mu c_t^1(c_v = c_v^1, S_t = s^k) + c_v^2 \cdot \mu c_t^1(c_v = c_v^2, S_t = s^k) \tag{9}$$

is the average cost conditioned by the states s^k .

Applying the optimum condition, we have:

$$\frac{d\,\pi_v}{d\,p_v} = 2b_v \cdot p_v + a_v - \bar{c_v} \cdot b_v = 0 \quad (10)$$

$$p_v = \frac{1}{2} \cdot (\bar{c_v} - \frac{a_v}{b_v}) \quad (11)$$

The price p_v obtained is the optimum price of V, denoted by p_v^o. It can be observed that p_v^o is influenced by the state of the source S_t. The state of the source influences the bi-dimensional distribution on $\Theta_1 \times \Theta_2$ and allows us to estimate the marginal distribution on the both costs.

Similar it is performed for obtaining the price for R :

$$p_r = \frac{1}{2} \cdot (\bar{c_r} - \frac{a_r}{b_r}) \quad (12)$$

The price p_r obtained is the optimum price of R, denoted by p_r^o.

In order to obtain non negative solutions for the prices p_v^o and p_r^o, the average costs $\bar{c_v}$ and $\bar{c_r}$ have to be greater than a_v/b_v, respectively a_r/b_r. As a consequence, the problem has useful solutions only for those states of the source S_t for which the above conditions are met.

The state of the source and the parameters of the econometric model of the demand influence both optimum prices.

Regarding the way for estimating the optimum prices (p_r^o, p_v^o) we shall made some observations. Firstly, the states of the source affect the optimum. If the probabilities regarding the state of the source are predicted for a given future time period $t+1$, then the model allows us to estimate the optimum (p_r^o, p_v^o) for the period $t+1$ conditioned by a state of the source. Secondly, the optimum is influenced by the econometric model of the demand. The demand parameters shall influence the optimum prices for V and R .

If the model is based only on the demand function of R , than normally $\bar{c_r} > \bar{c_v}$. The average costs difference between R and V allows us to analyze the opportunity to perform market research studies (demand functions) by R or both partners.

Another model is further presented. This model considers more complex demand functions for both partners. More information is received by the independent variables p_v and p_r within demand functions. In this case the optimum prices are the equilibrium prices estimated in the following game.

We shall consider the demand functions for V and R have the next form:

$$q_v(p_r, p_e) = a \cdot \Delta p_v + b \cdot \Delta p_r,\ q_r(p_v, p_r) = c \cdot \Delta p_v + d \cdot \Delta p_r \quad (13)$$

Where $\Delta p_i = p_i - \bar{p_i}, i \in \{V, R\}$, represents the deviation of price p_i from the average price $\bar{p_i}$ (Eichberger 1993). The demand functions q_v, q_r quantify the deviation from the average demand.

The payoff functions are:

$$\pi_v(p_v, p_r, c_v^i) = (a \cdot \Delta p_v + b \cdot \Delta p_r) \cdot (p_v - c_v^i) \quad (14)$$

$$\pi_r(p_v, p_r, c_r^i) = (c \cdot \Delta p_v + d \cdot \Delta p_r) \cdot (p_r - c_r^i) \quad (15)$$

$$i \in \{1,2\}.$$

In Bayesian equilibrium each player is supposed to choose a type contingent strategy, that is the decision function $p_v(\cdot)$ and $p_r(\cdot)$ respectively, which is the best response to the opponents' decision function.

First, taking $p_r(\cdot) = (p_r(c_r^1), p_r(c_r^2))$ as given and suppose that V has just learned that it has the cost c_v^1. Vendor's expected payoff can be written as:

$$\begin{aligned}
\pi_v(p_v, p_r, c_v^i) &= \pi_v(p_v(c_v^1), p_r(c_r^1)) \cdot \mu c_t^1(c_r^1/c_v = c_v^1, S_t = s^k) + \\
&+ \pi_v(p_v(c_v^1), p_r(c_r^2)) \cdot \mu c_t^1(c_r^2/c_v = c_v^1, S_t = s^k) = \\
&= (a \cdot (p_v(c_v^1) - \overline{p_v}) + \\
&+ b \cdot (p_r(c_r^1) - \overline{p_r}) \cdot (p_v(c_v^1) - c_v^1) \cdot \mu c_t^1(c_r^1/c_v = c_v^1, S_t = s^k) + \\
&+ (a \cdot (p_v(c_v^1) - \overline{p_v}) + \\
&+ b \cdot (p_r(c_r^2) - \overline{p_r}) \cdot (p_v(c_v^1) - c_v^1) \cdot \mu c_t^1(c_r^2/c_v = c_v^1, S_t = s^k)
\end{aligned} \quad (16)$$

Performing the steps from the previous model, we find the next four relations:

$$p_v(c_v^1) = \frac{b \cdot \overline{p_r} + a \cdot (\overline{p_v} - c_v^1)}{2 \cdot a} - \frac{b \cdot (p_r(c_r^1) \cdot \mu c_t^1(c_r^1/c_v = c_v^1, S_t = s^k) + p_r(c_r^2) \cdot \mu c_t^1(c_r^2/c_v = c_v^1, S_t = s^k))}{2 \cdot a \cdot (\mu c_t^1(c_r^1/c_v = c_v^1, S_t = s^k) + \mu c_t^1(c_r^2/c_v = c_v^1, S_t = s^k))} \quad (17)$$

$$p_v(c_v^2) = \frac{b \cdot \overline{p_r} + a \cdot (\overline{p_v} - c_v^2)}{2 \cdot a} - \frac{b \cdot (p_r(c_r^1) \cdot \mu c_t^1(c_r^1/c_v = c_v^2, S_t = s^k) + p_r(c_r^2) \cdot \mu c_t^1(c_r^2/c_v = c_v^2, S_t = s^k))}{2 \cdot a \cdot (\mu c_t^1(c_r^1/c_v = c_v^2, S_t = s^k) + \mu c_t^1(c_r^2/c_v = c_v^2, S_t = s^k))} \quad (18)$$

$$p_r(c_r^1) = \frac{d \cdot \overline{p_v} + c \cdot (\overline{p_r} - c_r^1)}{2 \cdot c} - \frac{d \cdot (p_v(c_v^1) \cdot \mu c_t^2(c_v^1/c_r = c_r^1, S_t = s^k) + p_v(c_v^2) \cdot \mu c_t^2(c_v^1/c_r = c_r^1, S_t = s^k))}{2 \cdot c \cdot (\mu c_t^2(c_v^1/c_r = c_r^1, S_t = s^k) + \mu c_t^2(c_v^1/c_r = c_r^1, S_t = s^k))} \quad (19)$$

$$p_r(c_r^2) = \frac{d \cdot \overline{p_v} + c \cdot (\overline{p_r} - c_r^2)}{2 \cdot c} - \frac{d \cdot (p_v(c_v^1) \cdot \mu c_t^2(c_v^1/c_r = c_r^2, S_t = s^k) + p_v(c_v^2) \cdot \mu c_t^2(c_v^1/c_r = c_r^2, S_t = s^k))}{2 \cdot c \cdot (\mu c_t^2(c_v^1/c_r = c_r^2, S_t = s^k) + \mu c_t^2(c_v^1/c_r = c_r^2, S_t = s^k))} \quad (20)$$

The relations 16, 17, 18 and 19 form a linear system of equations where the unknowns are $p_v(c_v^1), p_v(c_v^2), p_r(c_r^1), p_r(c_r^2)$. Solving the system, we shall obtain the equilibrium prices for V and R, denoted by: $p_v^e(c_v^1), p_v^e(c_v^2), p_r^e(c_r^1), p_r^e(c_r^2)$. The equilibrium price for V and R depends on their own cost, but also on the price and the cost the partner has. Analog, the prediction for the period $t + 1$ can be made.

CONCLUSIONS

The model can be used by all partners within a supply chain to calculate equilibrium prices for situations where internal and external information related to the supply chain is unknown. This information is very important while performing supply chain negotiations and establishing common strategies.

We do not consider information sharing for granted, but we do not consider that information sharing is sufficient for any player within the supply chain. The external information is not considered by most of the researchers. Another key element of our model is the fact that it is able to consider also the uncertainties available inside the supply chain.

We shall continue our research regarding this problem of integrating external information regarding supply chains using probability conditional distributions with a simulation model. For managers is very important to create scenarios regarding future activities. Most of models do not consider the uncertainty which influences supply chains. This simulation becomes though very important.

The model can be extended to the case in which there are more competing retailers instead of a single retailer. In this case, the vendor can calculate an equilibrium price for each retailer, being able to negotiate the price using specific information for each retailer.

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